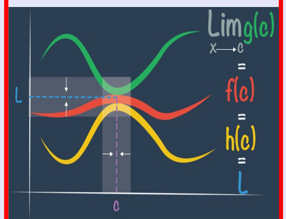
Math 261

Spring 2023

Lecture 26



Verify that $y = \cos x - 3\sin x$ satisfies

$$y''' + y'' + y' + y = 0. \checkmark$$

$$y = \cancel{\cos x} - \cancel{3\sin x}$$

$$y' = -\cancel{\sin x} - \cancel{3\cos x}$$

$$y'' = -\cancel{\cos x} + \cancel{3\sin x}$$

$$y''' = \cancel{\sin x} + \cancel{3\cos x}$$

$$y''' + y'' + y' + y = 0 \checkmark$$

Find $\frac{d^2y}{dx^2}$ if $\sin y + \cos x = 1$

$$\frac{d}{dx}[\sin y] + \frac{d}{dx}[\cos x] = \frac{d}{dx}[1]$$

$$\cos y \cdot \frac{dy}{dx} - \sin x = 0$$

$$\frac{dy}{dx} = \frac{\sin x}{\cos y}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right]$$

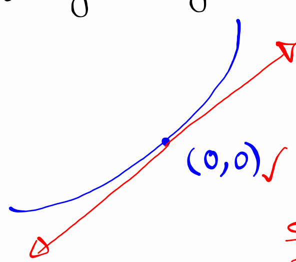
$$= \frac{d}{dx}\left[\frac{\sin x}{\cos y}\right] = \frac{\cos x \cdot \cos y - \sin x \cdot (-\sin y) \cdot \frac{dy}{dx}}{[\cos y]^2}$$

$$= \frac{\cos x \cos y + \sin x \sin y \cdot \frac{\sin x}{\cos y}}{\cos^2 y}$$

LCD = $\cos y$

$$\frac{d^2y}{dx^2} \to \boxed{= \frac{\cos x \cos^2 y + \sin^2 x \sin y}{\cos^3 y}}$$

Find equation of the Tan. line to the Curve given by $xy^2 = \sin(x+2y)$ at $(0,0)$.



$$m = \left.\frac{dy}{dx}\right|_{(0,0)}$$

$$\frac{d}{dx}[xy^2] = \frac{d}{dx}[\sin(x+2y)]$$

$$1 \cdot y^2 + x \cdot 2y \cdot \frac{dy}{dx} = \cos(x+2y) \cdot \left[1 + 2\frac{dy}{dx}\right]$$

evaluate at $(0,0) \to \frac{dy}{dx} = m$

$$\cancel{0^2 + 0 \cdot 2(0)} \cdot m = \cancel{\cos 0} \cdot [1 + 2m]$$

$$0 = 1 \cdot (1 + 2m)$$

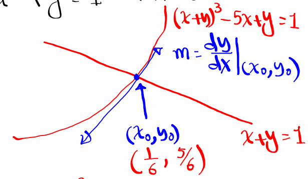
$$\boxed{m = \frac{-1}{2}}$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-1}{2}(x - 0)$$

$$\boxed{y = \frac{-1}{2}x}$$

Find equation of the tan. line at the point where the curve given by $(x+y)^3 - 5x + y = 1$ intersects the line $x+y=1$.



Intersection Point

$$\begin{cases}(x+y)^3-5x+y=1\\ x+y=1\end{cases} \qquad \begin{cases}1^3-5x+y=1\\ x+y=1\end{cases} \Rightarrow \begin{cases}-5x+y=0\\ x+y=1\end{cases}$$

$-6x=-1$

$\boxed{x=\frac{1}{6}}$

$y=1-\frac{1}{6}$

$\boxed{y=\frac{5}{6}}$

$(x+y)^3-5x+y=1$

$3\underbrace{(x+y)^2}_{1}\cdot\left(1+\frac{dy}{dx}\right)-5+\frac{dy}{dx}=0$

$3\left(1+\frac{dy}{dx}\right)-5+\frac{dy}{dx}=0$

$3+3\frac{dy}{dx}+\frac{dy}{dx}=5$

$4\frac{dy}{dx}=2 \rightarrow \boxed{\frac{dy}{dx}=\frac{1}{2}}$

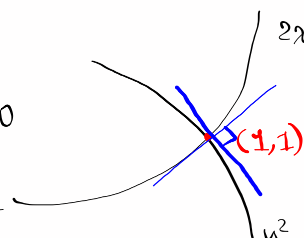
$m=\frac{1}{2}$

$y-y_1=m(x-x_1)$

$y-\frac{5}{6}=\frac{1}{2}\left(x-\frac{1}{6}\right)$

$\boxed{y=\quad}$

Show that $2x^2+3y^2=5$ and $y^2=x^3$ are orthogonal curves at $(1,1)$.



$2x^2+3y^2=5$

$2(1)^2+3(1)^2=5$ ✓

$y^2=x^3$

$1^2=1^3$ ✓

$2x^2+3y^2=5$

$4x+6y\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{-2x}{3y}$

$\left.\frac{dy}{dx}\right|_{(1,1)}=\frac{-2}{3}$

$2y\frac{dy}{dx}=3x^2$

$\frac{dy}{dx}=\frac{3x^2}{2y}$

$\left.\frac{dy}{dx}\right|_{(1,1)}=\frac{3}{2}$

$-\frac{2}{3}\cdot\frac{3}{2}=-1$

Product of slopes is -1.

tan. lines are perpendicular.

Curves are orthogonal at $(1,1)$.

Use linear approximation to estimate $\sqrt[3]{-8.25}$.

Use Cal. to find $\sqrt[3]{-8.25} \approx \boxed{-2.021}$

$f(x) = f(a) + f'(a)(x-a)$ where a is near

$f(x) = \sqrt[3]{x} \qquad a = -8$

$f(a) = \sqrt[3]{-8} = -2$

$f(x) = x^{1/3}$

$f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

$f'(a) = f'(-8) = \frac{1}{3\sqrt[3]{(-8)^2}} = \frac{1}{3 \cdot 4} = \frac{1}{12}$

$f(x) \approx f(a) + f'(a) \cdot (x-a)$

$\sqrt[3]{x} \approx -2 + \frac{1}{12}(x+8)$

Now plug in -8.25

$\sqrt[3]{-8.25} \approx -2 + \frac{1}{12}(-8.25+8) = -2 + \frac{1}{12}(-.25)$

$= -2 - \frac{1}{12} \cdot \frac{1}{4}$

$= -2 - \frac{1}{48}$

$= \frac{-97}{48} \approx \boxed{-2.021}$

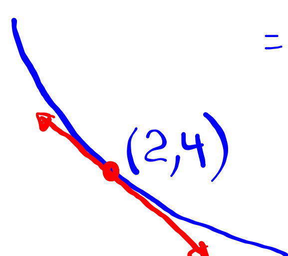
Class QZ 7

Find equation of the tan. line to the curve given by $xy = 8$ at $(2,4)$.

$xy = 8 \qquad (2)(4) = 8$ ✓

$y = \frac{8}{x} \qquad \frac{dy}{dx} = \frac{-8}{x^2}$

$m = \left.\frac{dy}{dx}\right|_{(2,4)} = \frac{-8}{2^2} = -2$



$m = \left.\frac{dy}{dx}\right|_{(2,4)}$

$y - y_1 = m(x - x_1)$

$y - 4 = -2(x - 2)$

$\boxed{y = -2x + 8}$